Laser Doppler Velocimetry

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Abstract: *Laser Doppler Velocimetry (LDV) can be used in measurement of flow rate as well as velocity profile in transparent flow channel. Relation between the frequency shift (f) and the velocity (v) was f=0.3245v. The sine of the beam angle is also directly proportional to the frequency. In addition to this, this experiment had also proved that LDV could use to determine the sizes and densities of particles in flow channel.*

Introduction:

As heat transfers are getting more and more important everyday, the study of fluid flow had became very crucial. In order to map out the velocity profiles or measuring the speed of the fluid with out disturbance, Doppler Velocimetry had been developed.

Laser Doppler Velocimetry "technique was first developed in 1964 by Yeh and Cummins who employed it to measure laminar water flow." (Lab, 1) As equipments that provided higher precisions had invented, the Doppler Velocimetry method had become more and more accurate and reliable.

The purpose of this experiment is to study the principle of Laser Doppler Velocimetry and its application on velocity profile mapping and particle sizes measurement.

Methods and Theories:

The setup of the experiment is as follow:

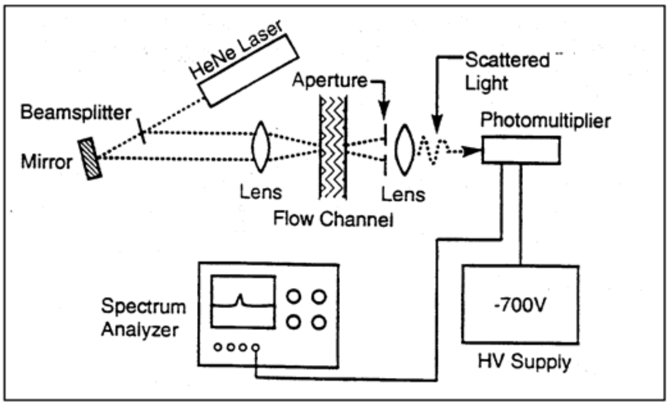


Fig 1: Optical System Setup (Lab, 2)

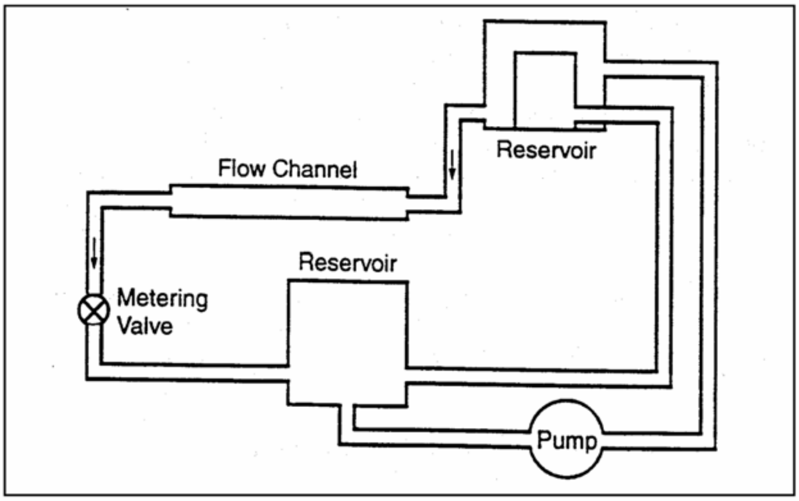


Fig 2: Flow System Setup

The idea is to use a beam splitter to divided the He-Ne Laser, converge the two beams at the center of the flow channel. As the flow particles pass by, each beam will scattered differently. With the different in frequencies, one can relate the velocity to it.

With this experimental setup, one would vary the flow rate, position of the beam crossing, beam angle and fluid content, in order to study the varies effects.

There are two different explanations for Laser Doppler Velocimetry, and both of them are equally valid and come down to the same result.

One of the interpretations of the Velocimetry is to have a beam splitter to split the beam in to two, and pass through the fluid channel at the same point in two different angles (direction). As the photon hit a moving particle, due to effect of Rayleigh scattering (Yeh and Cummins, 176), the wavelength of the photon will be shifted according to the Doppler shift relations. The deviation is as follow:

For Regular Doppler shift:

With the condition of v<<c

$$\frac{\lambda'}{\lambda} = 1 - \frac{v}{c}$$ (Pedrotti and Pedrotti, 184)

Where:

λ'=Shifted Wavelength

λ= Incident Wavelength

v=Observer-Source Relative Velocity (Particle Velocity in this case)

c=Speed of light

Consider there are two beams with same wavelength were used in this experiment (Fig 3)

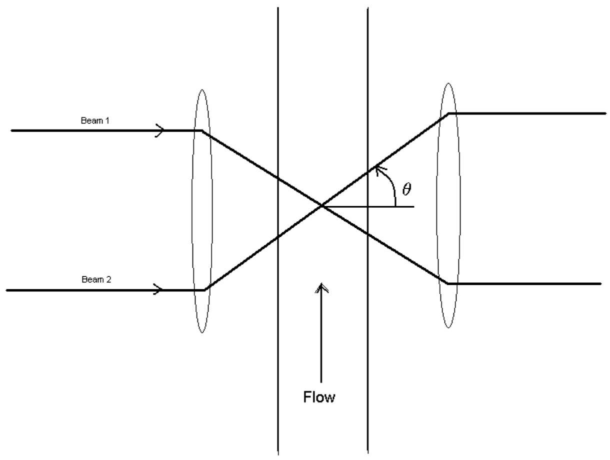


Fig 3: Beams incident

For Beam 1:

$$\frac{\vec{v}}{c} = \frac{v\sin\theta}{c} = 1 - \frac{\lambda_1'}{\lambda}$$
$$= \frac{\lambda - \lambda_1'}{\lambda} = \frac{\Delta\lambda_1}{\lambda}$$
$$\Rightarrow \Delta\lambda_1 = v\sin\theta \times \frac{\lambda}{c} = \frac{v\sin\theta}{\upsilon}$$
$$\Rightarrow \Delta\upsilon_1 = \frac{v\sin\theta}{\lambda}$$

Apply the same argument on Beam 2

$$\Delta\upsilon_{Total} = f = \Delta\upsilon_1 + \Delta\upsilon_2 = \frac{2v\sin\theta}{\lambda} \qquad \text{(Equation 1)}$$

Due to both incoming beam and outgoing beam are travelling in air. However, the actual medium measuring is water; one might need to add a correction factor related to the relative index of reflection to the equation according to Snell's law:

From Snell's Law

$$\sin\theta_{water} = \frac{n_{water}}{n_{air}}\sin\theta_{air}$$
$$\Theta\ n_{air} \approx 1 \qquad \text{(Equation 1')}$$
$$\therefore \sin\theta_{water} = n_{water}\sin\theta_{air}$$
$$\therefore f = \frac{2vn_{water}\sin\theta_{air}}{\lambda}$$

Another interpretation is base on the real fringe system being created in the cross over beam waist. Which is show is the below figure (Fig 4):

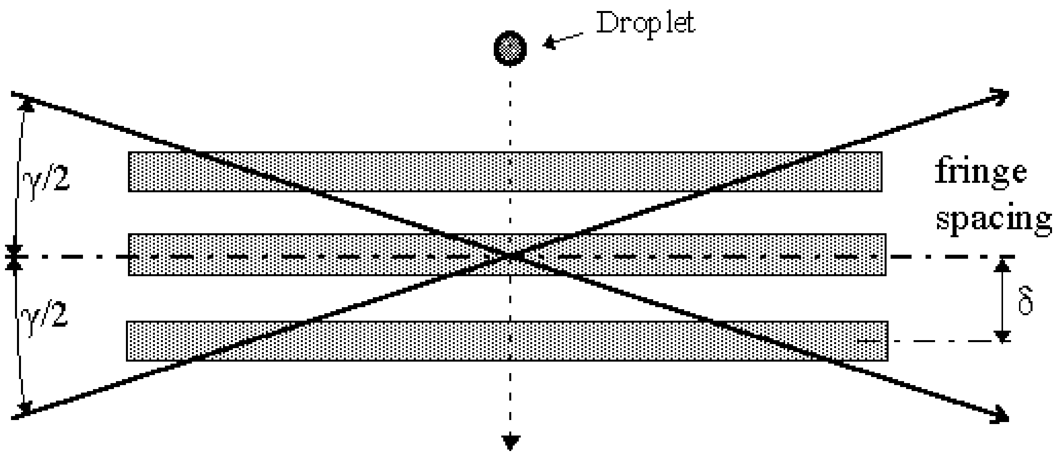


Fig 4: Real Fringe System (Sjoboen, Banks, and Petersen)

As the two beams cross over, an interference pattern will occur. Consider they have the exact same wavelength; the fringe will have equal spacing. The distance between the fringes spacing (δ) is given by following equation:

$$\delta = \frac{\lambda}{2\sin(\gamma/2)}$$ (Sjoboen, Banks, and Petersen)

As the droplet particle passes through the crossing area, the light frequency will be modulated by the velocity and the fringes. Therefore, the frequency shift (beat frequency) will be given by the quotient of the two. This is:

$$f = \frac{v}{\delta} = \frac{2v\sin(\gamma/2)}{\lambda} = equation\_1$$ (Sjoboen, Banks, and Petersen)

There were four part of this experiment; first, one would determine the relation between the maximum velocity and the scattering frequency. By determining the Flow rate of the water, with the use of stopwatch and measuring cylinder, one can also determine the average velocity of the flow using the principle of continuity.

$$v_{av} = Q/A$$ (Chang, 12)

Consider the flow channel is in square shape, one can consider it as plane Poiseuille flow. And the relationship between average velocity and the maximum velocity (which occur at the centre of the channel) are:

$$v_{av} = \frac{2}{y_0}\int_{-y_0}^{y_0} v\,dy = \frac{2}{3}v_{\max}$$ (Chang, 32)

With these two equations, one can deviate the relationship between flow rate and the maximum velocity, in a given cross section area.

$$v_{\max} = \frac{3Q}{2A}$$ (Equation 2)

Secondly, as the relationship between the shift frequency and the maximum velocity had established, a flow velocity profile can be map out by moving the beams crossing point in the channel.

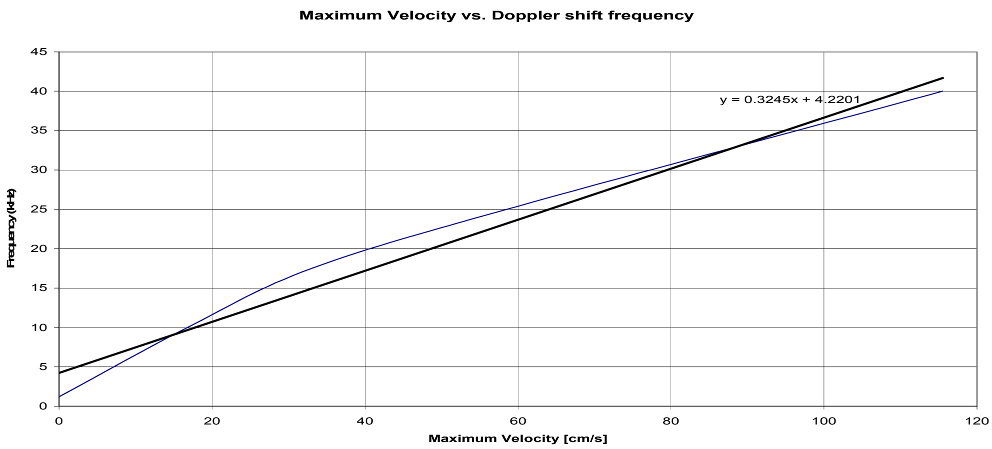
In the third part, one would evaluate the effect on the beam angle θ. In addition, at the end, one would use this technique to find out the size of milk powder particles, by relating the Doppler shift created by water and the milk particles.

Experimental Results:

In the first part, one had determine the Doppler shift frequency and its corresponding flow rate, since the cross-section area is known accurately (1.000±0.001cm by 1.000±0.001cm))

Flow Rate (L/min)	Frequency (kHz)	Average Velocity (U av = Q/A) [cm/s]	Maximum Velocity (U max = 3/2 U av) [cm/s]
0	0	0	0
0.67	10	11.16667	16.75
1.32	17.5	22	33
2.34	25	39	58.5
4.62	40	77	115.5

Table 1: Experimental results: Flow Rate vs. Frequency



Graph 1: Doppler Shift vs. Maximum Velocity

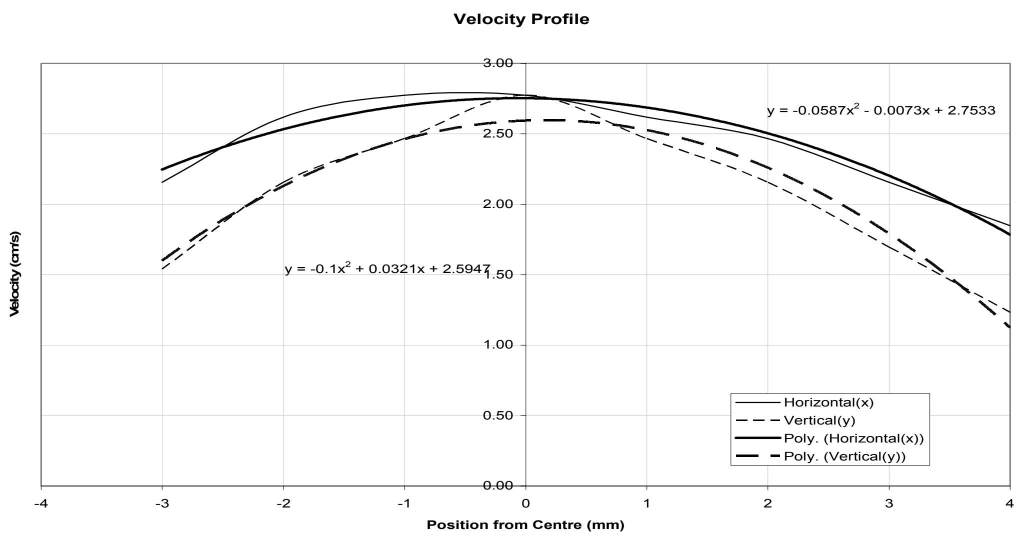
The angle was measured to be 8.06° (with simple trigonometry), by applying to equation 1', the coefficient should equal to 2n sin (θ)/λ = 0.2998 (Assuming Refractive index of water = 1.33). Which the experimental value is about 8.2% different compare to the theatrical, this could be a result of the impurity of the water. However, linearity had shown on the graph and it is pretty evidence that the theoretical prediction was correct.

As the relationship between Doppler shift frequency and the speed at the center point had been develop, one can now map out the velocity profile of the flow channel. Following are the result of measurement

Horizontal (x) coordinate (mm)	Frequency (Hz)	Corresponding Velocity (cm/s)	Vertical (y) coordinate (mm)	Frequency (Hz)	Corresponding Velocity (cm/s)
9	700	2.16	17	500	1.54
10	850	2.62	18	700	2.16
11	900	2.77	19	800	2.47
12	900	2.77	20	900	2.77
13	850	2.62	21	800	2.47
14	800	2.47	22	700	2.16
15	700	2.16	23	550	1.69
16	600	1.85	24	400	1.23

Table 2: Velocity Profile

In addition, if one graphs the above data, following graph (graph 2) will produced:



Graph 2: Velocity profile in flow channel

Part three; one had considered the effect of the beam angles on the velocity value. The following results were gathered:

Beam Angle (°)	Shift Frequency (kHz)	Sin θ
9.19	1.25	0.159
11.12	1.5	0.193

Table 3: Beam angle vs. Shift Frequency

In addition, at the end, one had added milk powder particles into the water, experimental result that had shown different Doppler shift frequency appeared. Results are as shown below (Fig 3):

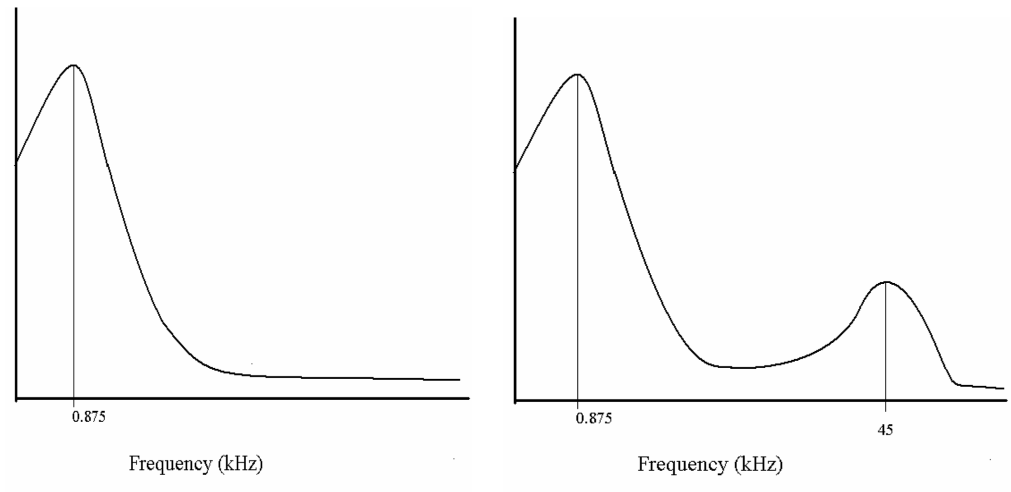


Fig 3: Sketch of Spectrum Analyzer Display (Left: Only Water, Right: With milk particles)

As the velocity of the fluid increase or decrease, the pattern will compress or expand respectively. It is evidence that the 45 kHz peak is corresponding to the milk particles. Moreover, as one change the concentration of milk particle, the height of the signal get higher in the 45 kHz peak. With these values, one shall be able to find out the size of the milk particles. Further discussion will explain in the next part of the report.

Discussion:

As one vary the velocity of the flow (by varying the flow rate), a linear relationship was observed between the velocity and the shift frequency. In addition, the slope given in the experimental results is only 8.2% different compare to theoretical. Therefore, the prediction of the theory was correct.

With the known values of frequency that corresponding to different flow velocities, a velocity profile can be map out by probing different position in the pipe. The result gather in this experiment mapped out two parabolas which corresponding to vertical and horizontal position. This result matches the Navier-Stokes equation prediction (Chang, 31-32).

With this technique, one can easily map out the velocity accurately without disturbing the flow itself. This technique is perfect for Laminar flow; however, for flow with high Reynold's number, turbulent flow will occur (Chang, 31). Therefore, velocity profile is no longer time independent. Therefore, this technique may not be used.

In terms of beam angles, according to equation 1, there should be a linear relationship between the frequency shift and the sine of the angle. Referring to the data, the ratio between the sine of the angles and ratio between the two frequency works out to be 1.2 in both case. Therefore, the linear relationship was confirmed.

With Doppler Laser Velocimetry, one can also detect the sizes of particle in the flow. The amount of scattering should directly relate to the radius of the particle (Hodkinson, 839). Consider only milk particle were being use in this experiment, no comparison were able to found. However, if a known size particle were available, one can calibrate that with water, in order to find out the size of the milk particle.

In addition, as the observation shown, one should be able to use the same technique to find out the particle density by knowing the relation of the height of the peak.

Conclusion:

In conclusion, Laser Doppler Velocimetry (LDV) can be used measure flow rate and generate velocity profile accurately for Laminar flow. A relationship found between the Doppler shift frequency and the velocity is f=0.3245v, where f is the frequency and v is the velocity.

However, when the flow rate is too high, Turbulent flow will occur. Therefore, the time dependence will introduce to the velocity profile according to Navier-Stokes equations (Chang, 31). As a result, scattering effect will be disturbed and LDV will not be able to use.

In this experiment, a confirmation was found for the velocity profile in a square flow channel. The shape was found to be a parabola, which also match with the theoretical result from Navier-stokes equation (Chang, 31)

According to equation 1, frequency shift will also directly proportional to the sine of the beam angle. And such relationship was confirmed with experimental results.

Moreover, with correct calibration with size known particle in the flow, one can determine the size of other unknown particles as well as its concentration.

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